Absorption Corrections for a Four-Quadrant SuperX EDS Detector

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Consider a four-quadrant detector consisting of four circular active regions of area \( A_q \) each, placed symmetrically around the sample, as shown schematically in Fig. 1(a). The sample holder is inserted from the right, and the quadrants are 90° apart, oriented symmetrically at ±45° with respect to the primary tilt axis. The specimen tilt angles are labeled \( \alpha \) and \( \beta \), with positive angles corresponding to counterclockwise rotations. Each detector quadrant is located above the plane of the specimen and is tilted from the vertical plane by an angle \( \delta_d \); the angle between the line connecting the center of a quadrant \( D \) with the eucentric point \( S \) and the horizontal plane is labeled \( \alpha_d \) (Fig. 1(b)). The active circular regions have a radius of \( R_d \), and a distance to the eucentric point of \( r_d \); the detector opening angle is \( R = R_d/r_d \). When \( R \) is not negligible, as is the case for a SuperX detector, then the x-ray photon path length inside the sample becomes a function of the position on the detector where the photon hits; the absorption correction factor must thus involve an integration over the detector surface area.

The path length \( \tau \) inside the sample for an x-ray photon leaving \( P_0 \) and traveling towards the point \( P_1 \) on the detector can be expressed as a fraction \( s \) of the distance between \( P_0 \) and \( P_1 \):

\[
\tau (x_d, z_d) = s(x_d, z_d) \sqrt{x_d^2 + z_d^2 + r_d^2} \quad \text{with} \quad s(x_d, z_d) = -\frac{n \cdot P_0}{n \cdot (P_1 - P_0)},
\]

where \((x_d, z_d)\) are the in-plane detector coordinates of the point \( P_0 \). Using the quadrant numbering of Fig. 1(a), one can show that the thickness-integrated dimensionless absorption correction factor for quadrant \( i \) is given by:

\[
C_i(\mu t) = \left[ \frac{1}{\pi R^2} \int_D d\bar{x} d\bar{z} \frac{1 - e^{-\mu f_i(\bar{x}, \bar{z})}}{\mu t f_i(\bar{x}, \bar{z})} \right]^{-1},
\]

where

\[
f_i(\bar{x}, \bar{z}) = \frac{\sqrt{\bar{x}^2 + \bar{z}^2} + 1}{a_i \bar{x} + b_i \bar{z} + c_i},
\]

with

\[
\begin{align*}
a_i &= s_i \sin \alpha \cos \beta + c_i \sin \beta; \\
b_i &= \cos \delta_d \cos \alpha \cos \beta + \sin \delta_d (c_i \sin \alpha \cos \beta - s_i \sin \beta); \\
c_i &= \sin \alpha_d \cos \alpha \cos \beta - \cos \alpha_d (c_i \sin \alpha \cos \beta - s_i \sin \beta),
\end{align*}
\]

and \( \bar{x} = x/r_d \) and \( \bar{z} = y/r_d \). This integral poses no numerical difficulties, as the integrand is well behaved, even when the denominator of \( f_i \) vanishes (in which case the integrand equals unity). In the absence of sample tilt (\( \alpha = \beta = 0 \)), the expression for \( f_i \) reduces to

\[
f_i(\bar{x}, \bar{z}) = \frac{\sqrt{\bar{x}^2 + \bar{z}^2} + 1}{\cos \delta_d \bar{z} + \sin \alpha_d},
\]

which, in the limit of vanishing detector radius \( R_d \), becomes

\[
f_i = \frac{z}{\sin \alpha_d} = z \csc \alpha_d,
\]
i.e., the standard result for a small distant detector. Since the four quadrant detectors are located symmetrically with respect to the specimen we have the following symmetry relations between the correction factors:

\[ C_2(\alpha, \beta; \mu t) = C_1(\alpha, -\beta; \mu t), \quad C_3(\alpha, \beta; \mu t) = C_1(-\alpha, -\beta; \mu t), \quad C_4(\alpha, \beta; \mu t) = C_1(-\alpha, \beta; \mu t), \]

so that we only need to compute \( C_1(\alpha, \beta; \mu t) \) over the entire range of tilt angles. Fig. 2 shows a contour plot of the inverse correction factor \( 1/C_1(\alpha, \beta; \mu t) \) as a function of tilt angles \( (\alpha, \beta) \) and for the detector parameters indicated in the upper left corner.

References
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**Figure 1.** (a) Schematic representation of a double tilt sample holder and four quadrant detectors placed symmetrically around the specimen; (b) Schematic of the specimen with thickness \( t \), and a single circular quadrant detector

**Figure 2.** Contour plot of \( 1/C_1(\alpha, \beta; \mu t) \) for the detector parameters indicated in the upper left corner. The rectangle superimposed on the plot shows the symmetry relations between an arbitrary pair of angles \( (\alpha, \beta) \) and the corresponding points for the other three detector quadrants.